# A Simple Method to Calculate the Maximum Usable Frequency

Jonas R. Souza, Inez S. Batista, Renata G. D. F. Costa, Divisão de Aeronomia/INPE, Brazil



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**Abstract (Font: Arial Bold, 9)**

**Radio communication community has great interest to know the daily behavior of the maximum usable frequency (MUF) for its applications. The MUF is obviously dependent of the ionospheric F-layer critical frequency (foF2) and its corresponding altitude (hmF2). In this work, the parameters foF2 and hmF2 and a spherical geometry are used to calculate MUF considering and not considering the ionospheric refraction effects in the electromagnetic wave propagation. The values of foF2 and hmF2 were obtained from ionograms and from a Parameterized Regional Ionospheric Model (PARIM). We have also compared our calculated MUF values with experimental data for different solar flux levels and for different locations over the Brazilian sector. In general, the results present very good agreement with data recorded by digisondes-ionosondes. Maps of MUFs constructed using ionospheric peak parameters given by PARIM show coherent results.**

## Introduction

The MUF for a certain distance on the Earth's surface can be determined by multiplying the critical frequency of the layer in consideration by the corresponding M-factor. The mentioned distance is known as hop which is the distance on the Earth's surface between two locations where occurs the signal transmission and reception after the ionospheric reflection. A crude method used to calculate the M-factor is one that just considers a spherical geometry between Earth-ionosphere. Another well know procedure to obtain the M-factor for a conventional hop of 3000 km is using a standard ratio (MUF/foF2) for each virtual height as published by Piggott and Rawer, 1978. This procedure is more reliable than the first one, but it is necessary to know a well defined ionogram.

In this work, our main aim is to present a method to calculate M-factor for a hop of 3000 km using only foF2 and hmF2 obtained from physical or empirical models or also from satellite data, i.e., non-necessarily only from ionograms. To calculate the MUFs for different hops, we have used an algorithm developed by Loockwood, 1983 which is recommended by the International Telecommunication Union, ITU (www.itu.int).

## Method to calculate the MUF

The ionospheric MUF can be calculated by

$$MUF = M(D) foF2\,, \tag{1}$$

where *M(D)* is the M-factor for a hop equal to *D*.

A simple way to determine M-factor is using a spherical geometry model. The M-factor obtained with such geometry is given by

$$M(D) = \sec\left\{\arcsin\left(\frac{R_E \cos\varepsilon_1}{R_E + hmF2}\right)\right\}, \tag{2}$$

where $R_E$ is the Earth's radio and $\varepsilon_1$ is the elevation angle, as can be seen in Figure 1. This Figure illustrates the geometry used to calculate M-factor.

The procedure presented above is not accurate due to the ionosphere refraction. To improve the M-factor calculation, we have considered the ionospheric effects which cause a time delay in the signal propagation and, consequently, an error in M-factor values obtained by only considering spherical geometry. This means that M-factor must be calculated for a virtual hmF2 value which can be given by

$$h'mF2 = hmF2 + \Delta h\,, \tag{3}$$

where $\Delta h = \dfrac{40.3}{(foF2)^2} TEC'$ and *TEC'* is the total electron content below hmF2. Considering such ionospheric effect the new M-factor can be expressed as

$$M'(D) = \sec\left\{\arcsin\left(\frac{R_E \cos\varepsilon_2}{R_E + h'mF2}\right)\right\}. \tag{4}$$

Here, $\varepsilon_2$ is the elevation angle for a signal reflected at h'mF2. This methodology works for D = 3000 km. The M-factors, for hops lesser than 3000 km, were calculated by the algorithm published by Loockwood, 1983. The Lookwood's algorithm uses M(3000), foF2, and E-layer critical frequency (foE) as input parameters.

## Data and method validation

We compare our MUF results with observations collected over two equatorial stations São Luís (2.3°S; 44°W) and Fortaleza (4°S; 38°W) and also over a low-latitude location Cachoeira Paulista (22.5°S; 45°W) both on the Brazilian territory. The data were registered by digisondes, except for Fortaleza that was used an ionosonde. All data are representative of equinoctial conditions and for geomagnetically quiet periods.

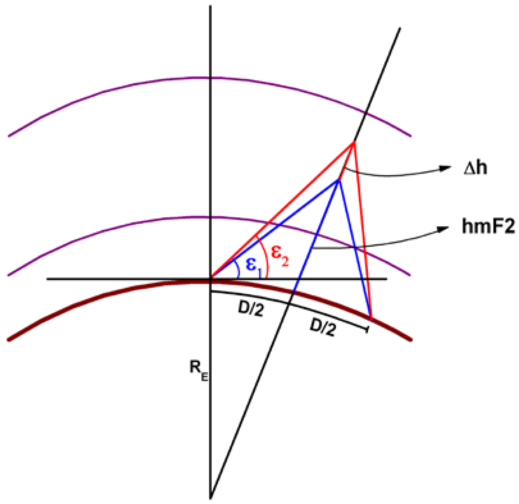


Figure 1: illustration of a spherical geometry used to calculate the M-factor.

To validate our method, the MUFs(3000) were calculated using foF2, hmF2 and the integrated vertical electron distribution (TEC') from ionograms.

Figure 2 shows the MUFs calculated without corrections of the ionospheric effects, i.e., using only spherical geometry (see solid blue dots), with ionospheric effect corrections (red dots) and experimental data (black dots). The results are for São Luís, Fortaleza and Cachoeira Paulsta. The first-line panels present the MUFs for solar minimum conditions and those ones in the second line are for solar maximum.

The MUF values calculated using the ionospheric effect corrections present good agreement with the data for both solar minimum and solar maximum conditions. The disagreements between the results using only spherical geometry, coherently, just confirm the need of the ionospheric effect corrections.

The MUF calculated for hops lesser than 3000 km were also validated. Figures 3 and 4 show MUFs for solar minimum and solar maximum respectively. Top panels are the results for hop = 200 km, middle panels are for hop = 600 km and the bottom panels present the results for hop = 1500 km. The MUFs calculated applying only the spherical geometry model show significant differences when compared with the ionogram data, except for hop = 200 km in which the differences basically disappear. The results are overestimated at the most of the times during both solar minimum and solar maximum periods. Since the M-factor tends to be equal 1 for small hops, the nice agreement for the lowest hop value was already expected. All MUF calculations, using the combination of our methodology with the Loockwood's method, present excellent agreement with the data. Such validation was analyzed for different solar flux levels, different hops and locations, as we can see in Figures 3 and 4.

**MUF maps over Brazil**

Since we have a well established method to determine MUFs, we decide to examine the performance of a Parameterized Regional Ionospheric Model, PARIM. This model was developed to calculate 3D electron density over Brazil and part of South America (Souza et al., 2010, 2013). Here, PARIM was run to obtain the values of foE, foF2 and hmF2 which are the standard inputs of our method.

Figure 5 and 6 show MUF maps constructed with inputs given by PARIM for two solar flux levels, solar minimum and solar maximum, respectively. The results are also representative of equinoctial conditions. The three columns present MUFs for hop of 600, 1500 and 3000 km, respectively. At each column there are results for 09, 15 and 21 Universal Time (UT). The magenta line crossing the maps is the position of the magnetic equator. The presence of the equatorial ionization anomaly is very clear for both solar minimum and solar maximum conditions, mainly during evening time.

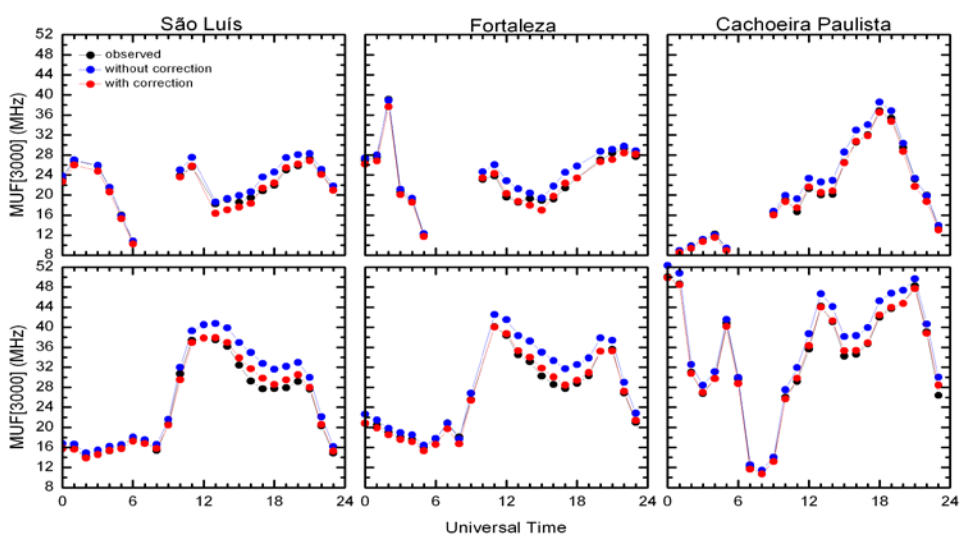


Figure 2: Diurnal variations of the MUFs calculated with and without corrections of the ionospheric effects and experimental data for solar minimum (top panels) and solar maximum periods (bottom panels).

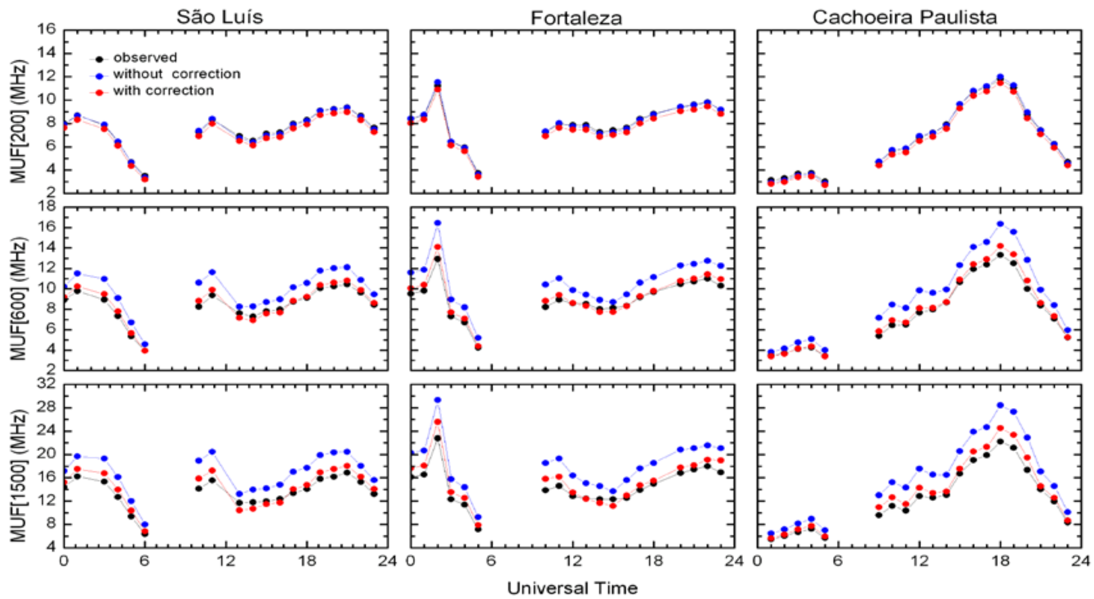


Figure 3: Diurnal variations of the MUFs calculated with and without corrections of the ionospheric effects and experimental data for solar minimum and for hops of 200 (first line), 600 (second line) and 1500 km (third line).

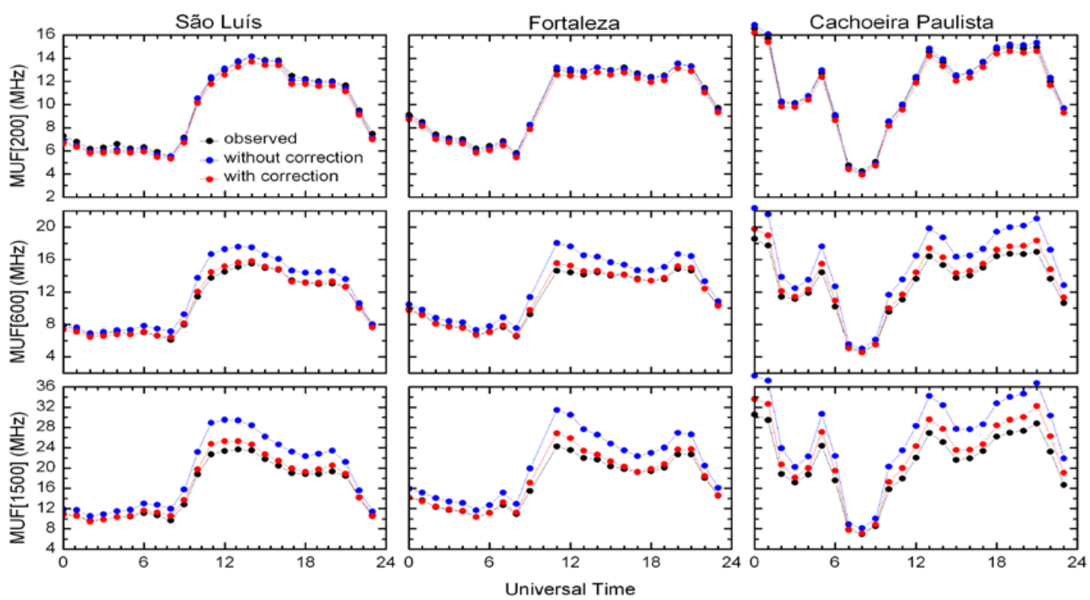


Figure 4: Same as Figure 3, but for solar maximum conditions.

One interesting point to be noticed occurs when we have strong equatorial ionization anomaly development, as for example during high solar activity as presented in Figure 6, it is not possible to establish communication in east-west direction with a frequency range near those from anomaly crests. Obviously, for locations below the anomaly crests the east-west communications are possible using such frequencies.

Unfortunately, we did not have enough experimental data available to do MUF maps to compare them with our results. A full validation, i.e., validation covering all Brazilian paces is necessary and it will be presented in a future publication. The uncertainties on the maps calculated using PARIM are in the uncertainties in the parameters foE, foF2 and hmF2 given by the PARIM itself. On the other hand, the values of foE, foF2 and hmF2 calculated for Conjugate Point Experiments

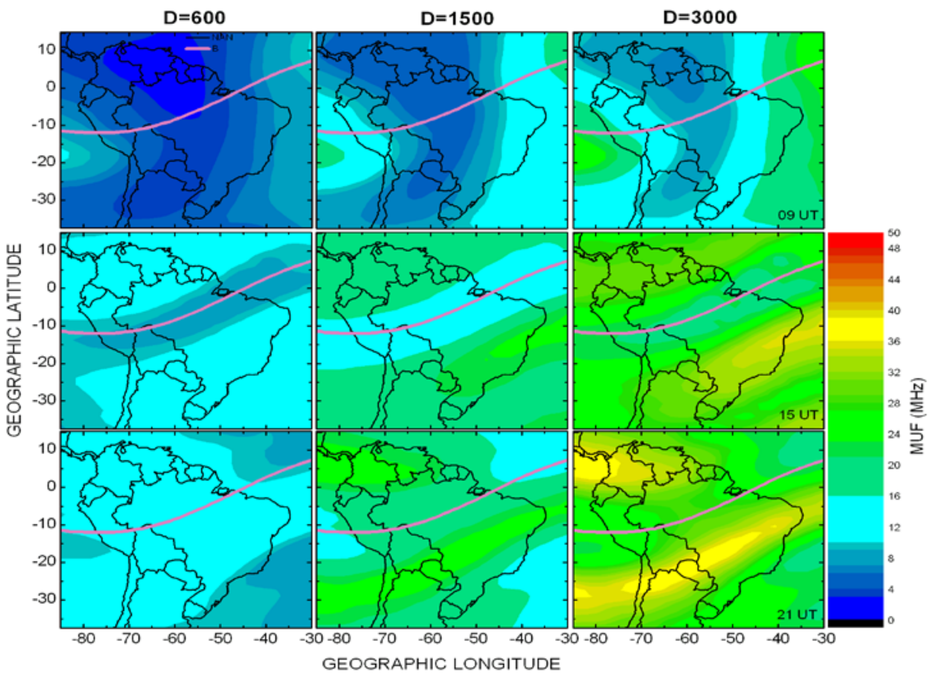


Figure 5: Maps of MUF calculated with foE, foF2 and hmF2 from PARIM for solar minimum conditions.

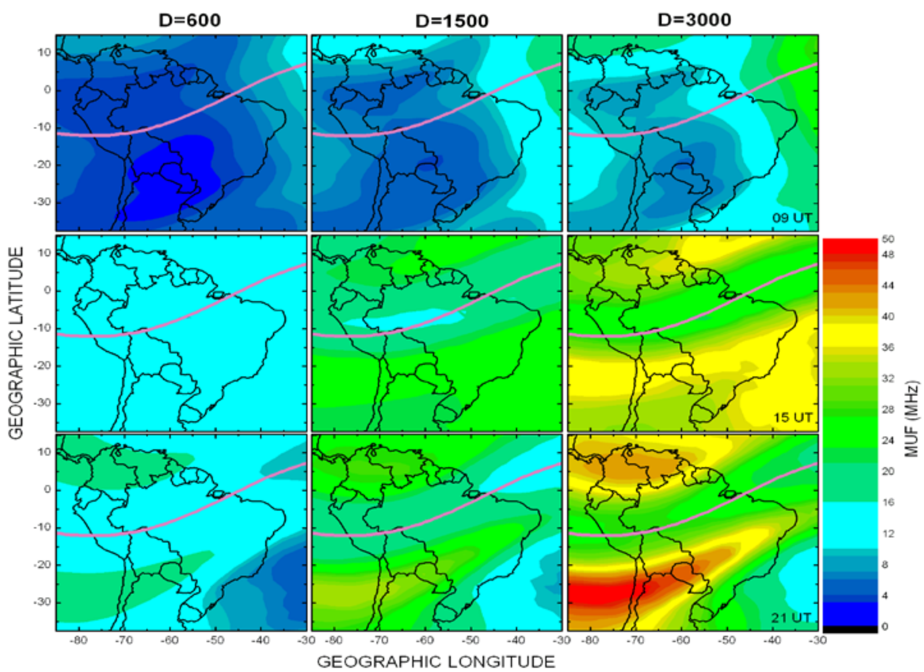


Figure 6: Same as Figure 5, but for solar maximum conditions.

(COPEX campaign) presented good agreement with data, as published by Souza et al. 2013.

**Conclusions**

The main conclusion of this work may be summarized as follows:

(i) We have described a method to calculate MUF using foE, foF2 and hmF2. The method can be easily coupled to physical, empirical or parameterized models to make MUF predictions.

(ii) The MUFs calculated by applying only spherical geometry showed overestimate values when compared with experimental data.

(iii) MUF calculated using spherical geometry and including the ionospheric effect corrections presents good agreement with data for a hop of 3000 km.

(iv) PARIM model has presented coherent predictions of MUFs for the Brazilian sector during both solar minimum and solar maximum conditions.

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